Stress Analysis of a Pressurized Pipe with an End Cap

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For questions, please fill out contact form

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Model Description

- Stress Analysis of a pressurized pipe with and end cup based on example question 2 documented in page 122 of the following book :
  - J. Souza, *Roark's Formulas For Stress And Strain-.pdf*. Accessed: Nov. 06, 2022. [Online]. Available: https://www.academia.edu/37205286/Roarks_Formulas_For_Stress_And_Strain_pdf
- PVC is selected as the material for the pipe.

Model Description

a. Inner radius – $0.9695\ in$

b. Outer radius – $1.1875\ in$

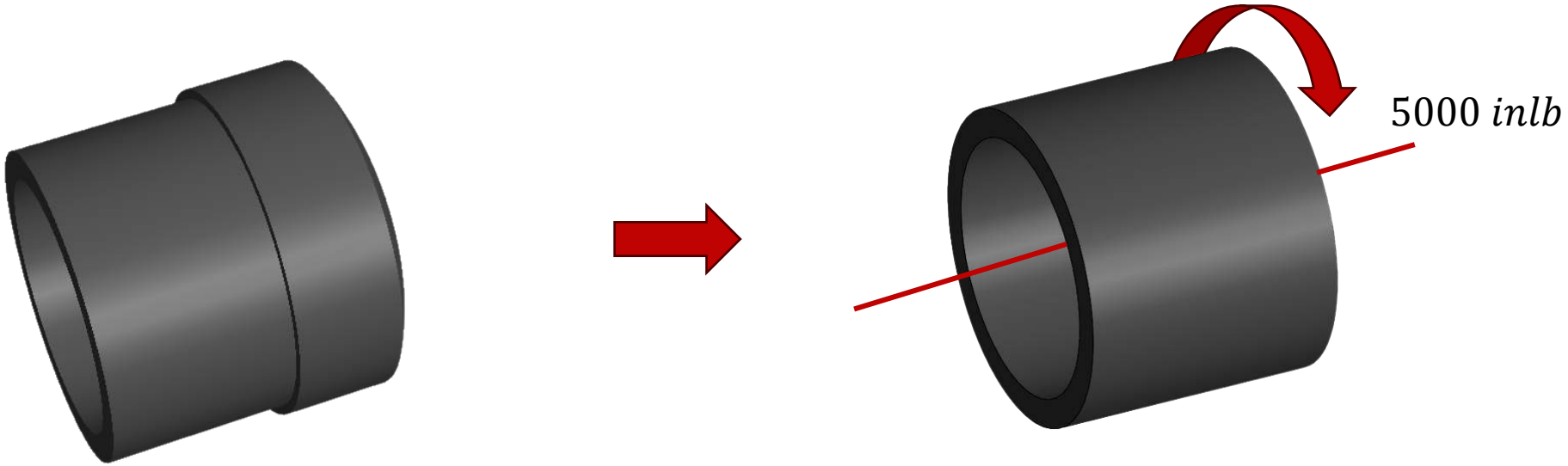
c. Cross sectional area of bore – $2.955\ in^2$

d. Cross sectional area of pipe wall – $1.475\ in^2$

e. Polar moment of inertial – $1.735\ in^4$

Model Description

- The case study (Pipe with end cap) is simplified as follows,
  - The end cap is replaced with a twisting moment (at end edge of pipe) which is caused by the end cap as mentioned in the example question.



Model Parameters

Entity	Type
Solver	Altair SimSolid
Version	2022.2.1
CPU	Intel(R) Core(TM) i7-9750H CPU @ 2.60GHz

FEA Entities	Type
Analysis Type	Structural linear
Unit System	lb, in, s

Analysis Assumptions and Limitations

- For the simulation, an acceleration has been calculated to produce the twisting moment of **5000 in-lb** and assumed resultant effect from acceleration is equal to the effect of the moment,

$$Twisting\ Moment = 5000\ lb\ in$$

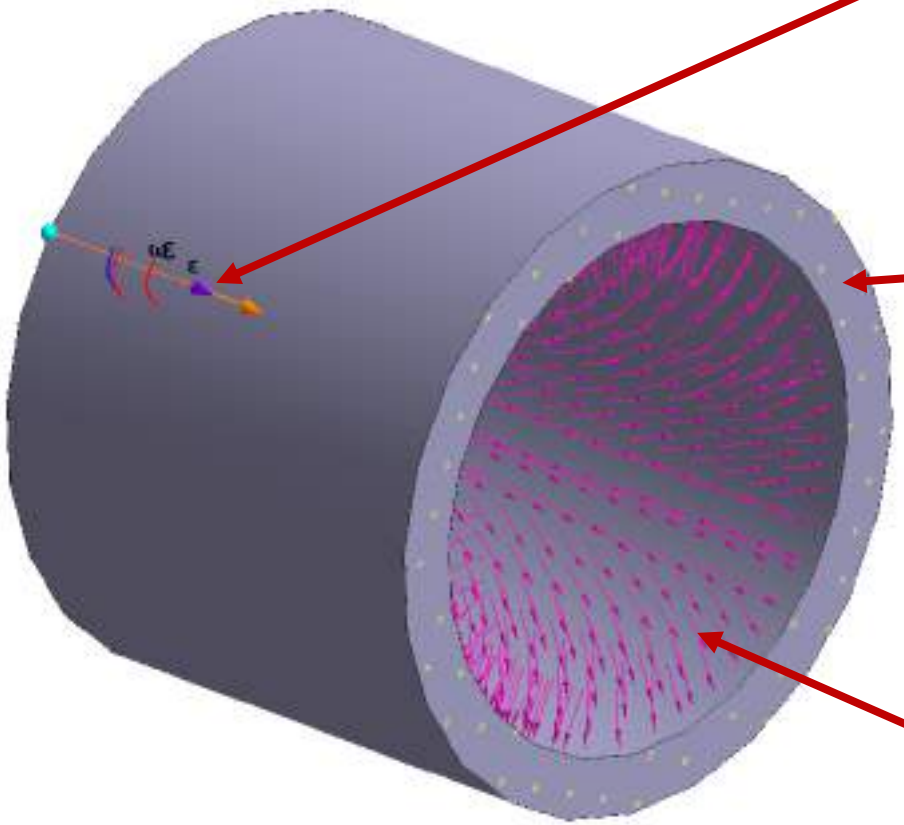
$$Polar\ Moment\ of\ inertia = 1.735\ in^4$$

Using $Torque = I\alpha$,

$$\alpha = \frac{5000}{1.735}$$

$$\alpha = 2881.84\ rads^{-2}$$

Analysis Setup



Acceleration
$2881.84\ rads^{-2}$

Immovable
Constraint

Internal pressure =
2000psi

Hand Calculations

- For a point on inner surface,

$$\sigma_x = \frac{pA_b}{A_w} = \frac{(2000)(2.955)}{1.475} = 4007 \text{ lb/in}^2$$

$$\sigma_y = p\frac{r_o^2 + r_i^2}{r_o^2 - r_i^2} = 2000\frac{1.1875^2 + 0.9695^2}{1.1875^2 - 0.9695^2} = 9996 \text{ lb/in}^2$$

$$\sigma_z = -p = -2000 \text{ lb/in}^2$$

$$\tau_{xy} = \frac{Tr_i}{J} = \frac{(5000)(0.9695)}{1.735} = 2794 \text{ lb/in}^2$$

$$\tau_{yz} = \tau_{zx} = 0$$

$$\begin{vmatrix} (\sigma_x - \sigma_p) & \tau_{xy} & \tau_{zx} \\ \tau_{xy} & (\sigma_y - \sigma_p) & \tau_{yz} \\ \tau_{zx} & \tau_{yz} & (\sigma_z - \sigma_p) \end{vmatrix} = 0$$

$$\sigma_p^3 - (4007 + 9996 - 2000)\sigma_p^2 + [(4007)(9996) + (9996)(-2000) + (-2000)(4007) - 2794^2 - 0 - 0]\sigma_p - [(4007)(9996)(-2000) + 2(2794)(0)(0) - (4007)(0^2) - (9996)(0^2) - (-2000)(2794^2)] = 0$$

or

$$\sigma_p^3 - 12.003(10^3)\sigma_p^2 + 4.2415(10^6)\sigma_p + 64.495(10^9) = 0$$

- Solving this give $\sigma_p = 11{,}100,\ 2906, \text{ and } -2000 \text{ lb/in}^2$,
- The maximum shear stress, $0.5[11{,}100 - (-2000)] = \mathbf{6550\ lb/in^2}$
- Detailed Hand calculations is mentioned in following book pages 122-124,

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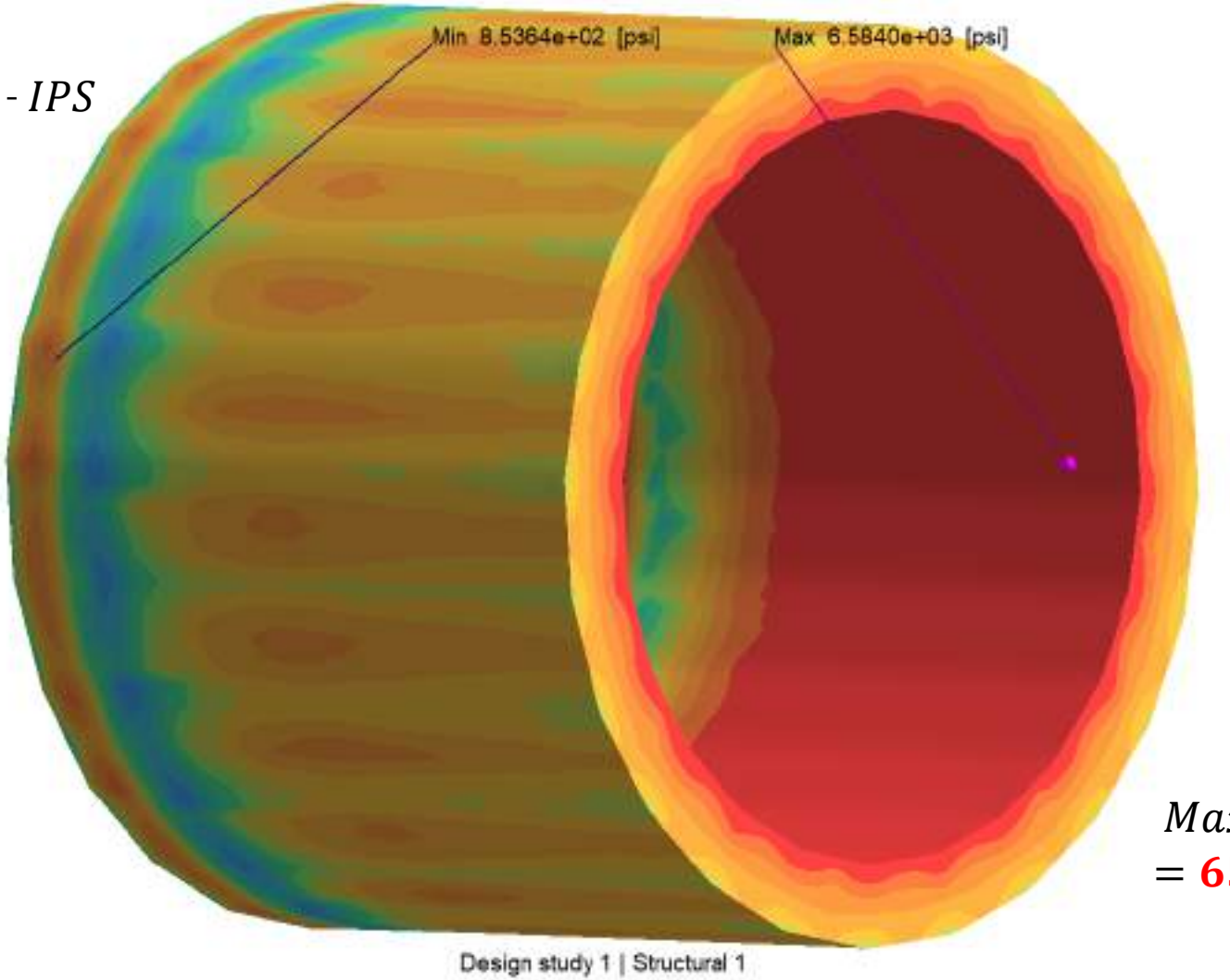
Analysis Results – Stresses

Units - IPS



$Maximum\ Shear\ Stress(Simulation) = \mathbf{6584\ lb/in^2}$

Analysis Results

- Maximum Shear Stress occur due to the twisting moment from the tightening end cap,

  Based on hand calculations – $6550\ lb/in^2$

  From the simulation - $6584\ lb/in^2$

- Error percentage = $\frac{(6584-6550)}{6550} * 100\%$

  $= 0.5191\ \%$

Conclusions

- Stress analysis of a Pressurized Pipe With An End Cap conducted using Altair SIMSOLID based on the book listed in slide 2.
- Results of the simulation correlate well to the expected hand calculation value.