Bending of an annular plate

For questions, please use contact form

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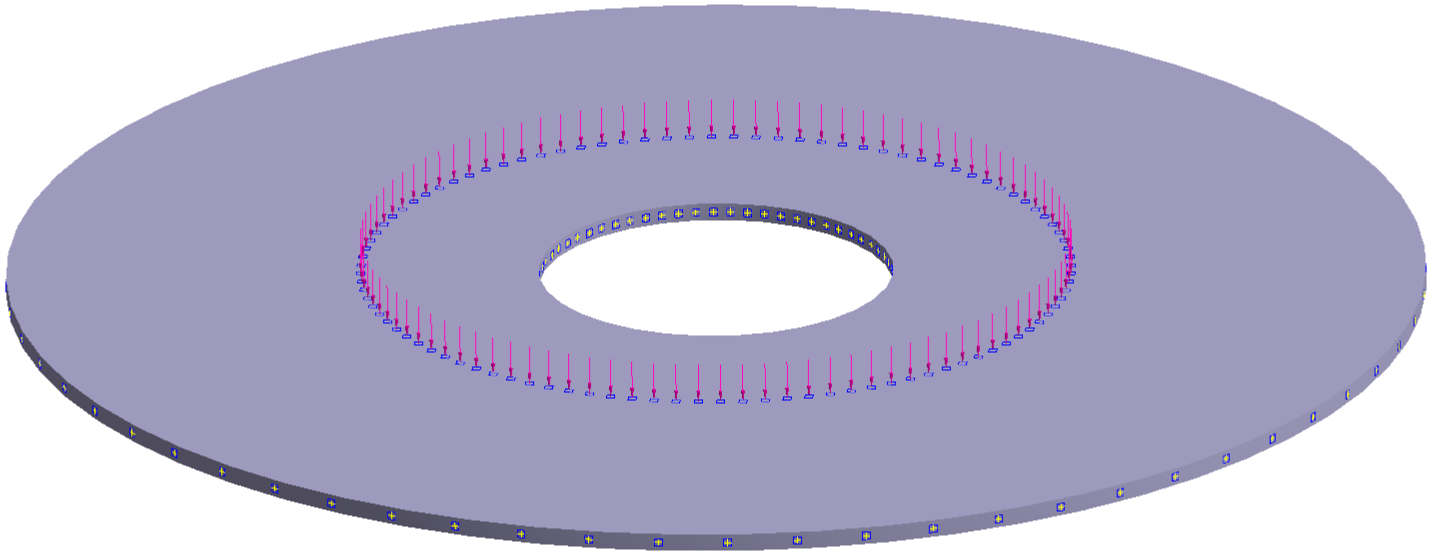
Model Description

- Deflection analysis of a beam is based on example question 2 documented in page 430 of the following book :
  - J. Souza, *Roark's Formulas For Stress And Strain-.pdf*. [Online]. Available: https://www.academia.edu/37205286/Roarks_Formulas_For_Stress_And_Strain_pdf

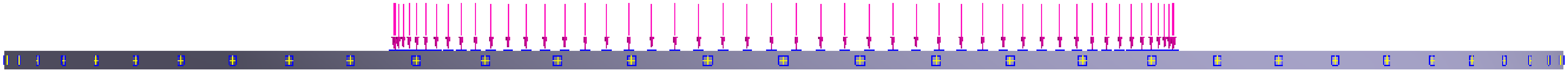


Model Description

Young’s modulus – 68.947 GPa

Density – 1000 kg/m3

Poison’s ratio – 0.3



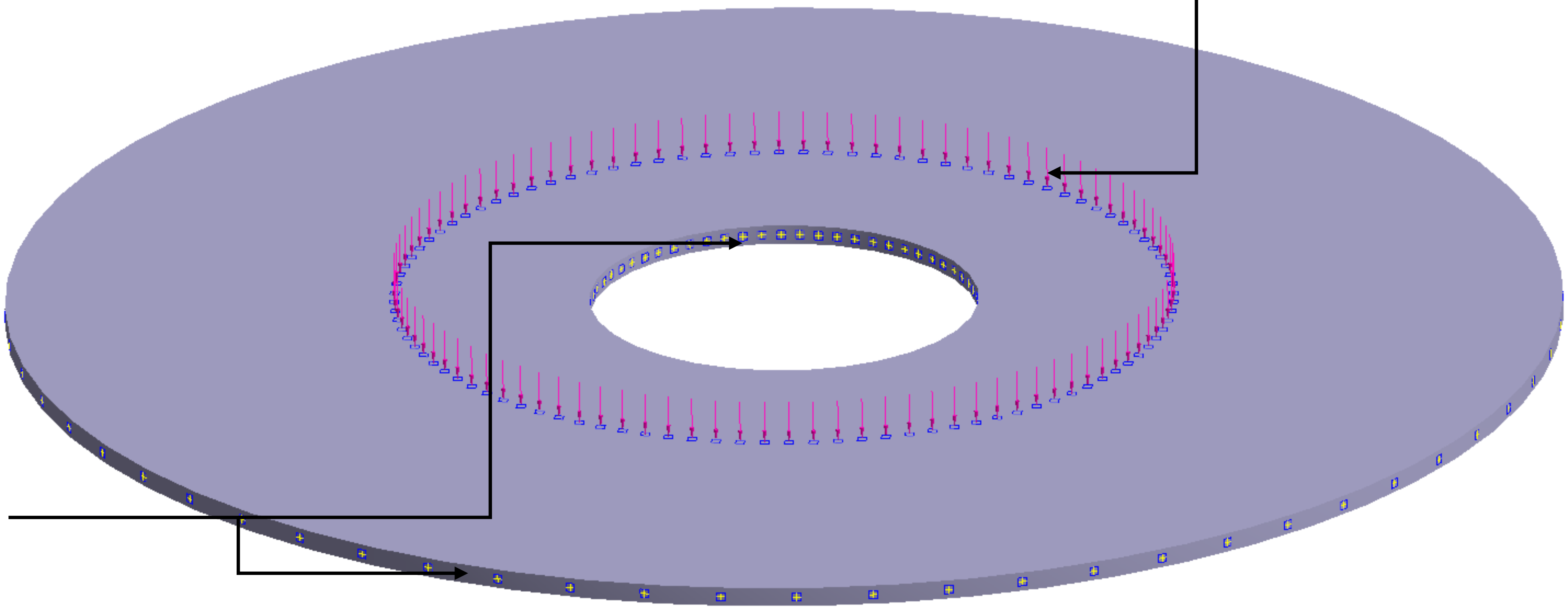
Model Parameters

Entity	Type
Solver	Altair SimSoild
Version	2022.2.1
CPU	Intel(R) Core(TM) i7-9750H CPU @ 2.60GHz

FEA Entities	Type
Analysis Type	Structural linear
Unit System	lb, in, s

Analysis Setup

Annular line load
- 40 lbf/in
- Downward



Simple supports
- Inner edge
- Outer edge

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Analysis Assumptions and Limitations

- Thickness of the plate is assumed as 0.5in.

Hand Calculations

For this calculation, case 1c of the table 11.2 should be referred.

For this problem,

a – 20 in, b – 5 in, r0 – 10 in, w – 40 lb/in

E – 10^7 psi, ν – 0.3

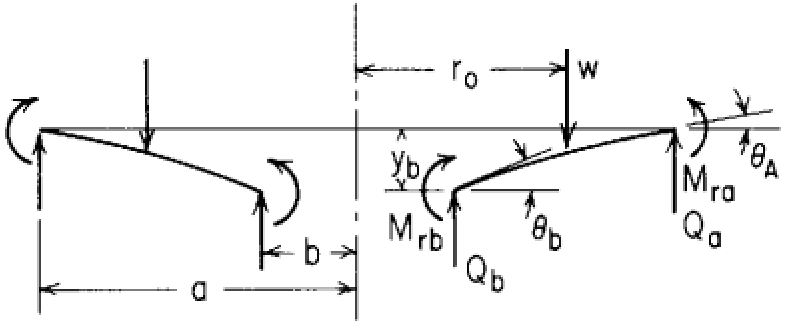

Figure – From above book

Initially following coefficients are determin

$$C_1 = \frac{1+\nu}{2}\frac{b}{a}\ln\frac{a}{b} + \frac{1-\nu}{4}\left(\frac{a}{b}-\frac{b}{a}\right) = 0.881523$$

$$C_3 = \frac{b}{4a}\left\{\left[\left(\frac{b}{a}\right)^2+1\right]\ln\frac{a}{b} + \left(\frac{b}{a}\right)^2 - 1\right\} = 0.03346$$

Hand Calculations

$$C_7 = \frac{1}{2}(1 - \nu^2)\left(\frac{a}{b} - \frac{b}{a}\right) = 1.70625$$

$$C_9 = \frac{b}{a}\left\{\frac{1+\nu}{2}\ln\frac{a}{b} - \frac{1-\nu}{4}\left[1 - \left(\frac{b}{a}\right)^2\right]\right\} = 0.266288$$

$$L_3 = \frac{r0}{4a}\left\{\left[\left(\frac{r0}{a}\right)^2 + 1\right]\ln\frac{a}{r0} + \left(\frac{r0}{a}\right)^2 - 1\right\} = 0.014554$$

$$L_9 = \frac{r0}{a}\left\{\frac{1+\nu}{2}\ln\frac{a}{r0} + \frac{1-\nu}{4}\left[1 - \left(\frac{r0}{a}\right)^2\right]\right\} = 0.290898$$

Hand Calculations

From case 1c,

$y_b = 0, M_{rb} = 0$

$$\vartheta_b = \frac{-wa^2}{D}\frac{C_3L_9 - C_9L_3}{C_1C_9 - C_3C_7} = \frac{-527.8}{D} rad$$

$$Q_b = w\frac{C_1L_9 - C_7L_3}{C_1C_9 - C_3C_7} = 52.15\ lb/in$$

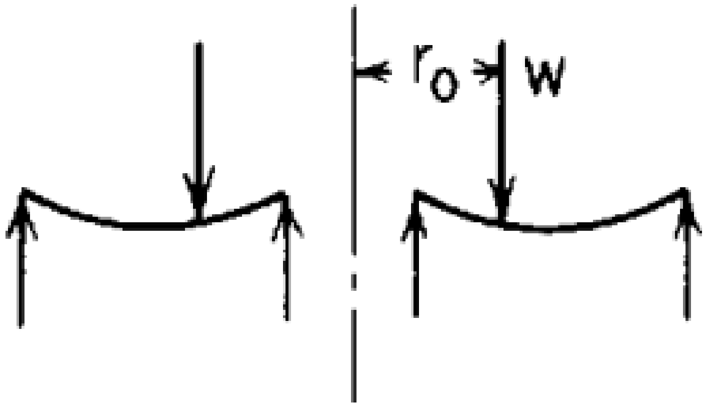

Figure – From above book

Substituting above values to following equation,

$$y = y_b + \vartheta_b r F_1 + M_{rb}\frac{r^2}{D}F_2 + Q_b\frac{r^3}{D}F_3 - w\frac{r^3}{D}G_3$$

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Hand Calculations

Therefore,

$$y = 0 - \frac{527.8r}{D}F_1 + 0 + \frac{52.15r^3}{D}F_3 - \frac{40r^3}{D}G_3$$

Expressions for F_1, F_3 and G_3 are as follows,

$$F_1 = \frac{1+\nu}{2}\frac{b}{r}\ln\frac{r}{b} + \frac{1-\nu}{4}\left(\frac{r}{b} - \frac{b}{r}\right)$$

$$F_3 = \frac{b}{4r}\left\{\left[\left(\frac{b}{r}\right)^2 + 1\right]\ln\frac{r}{b} + \left(\frac{b}{r}\right)^2 - 1\right\}$$

$$G_3 = \frac{r0}{4r}\left\{\left[\left(\frac{r0}{r}\right)^2 + 1\right]\ln\frac{r}{r0} + \left(\frac{r0}{r}\right)^2 - 1\right\}$$

To reduce the complexity of the calculations, evaluate y at specific values of r, then an excellent approximation to the maximum deflection can be obtained.

Hand Calculations

From the table mentioned in the page 431 of the book, maximum deflection is located at a radius near 11.25 in and has a value approximately,

$$y(D) = \frac{-1900}{D}$$

Here,

$$D = \frac{Et^3}{12(1 - \nu^2)}$$

Therefore,

$$Maximum\ deflection = 0.01656$$

Hand Calculations

Maximum bending stress is developed at a radius of 10 in.

$$M_r = \vartheta_b \frac{D}{r} F_7 + M_{rb} F_8 + Q_b r F_9 - w r G_9$$

For this,

$$F_7 = 0.6825$$
$$F_9 = 0.290898$$

Hence,

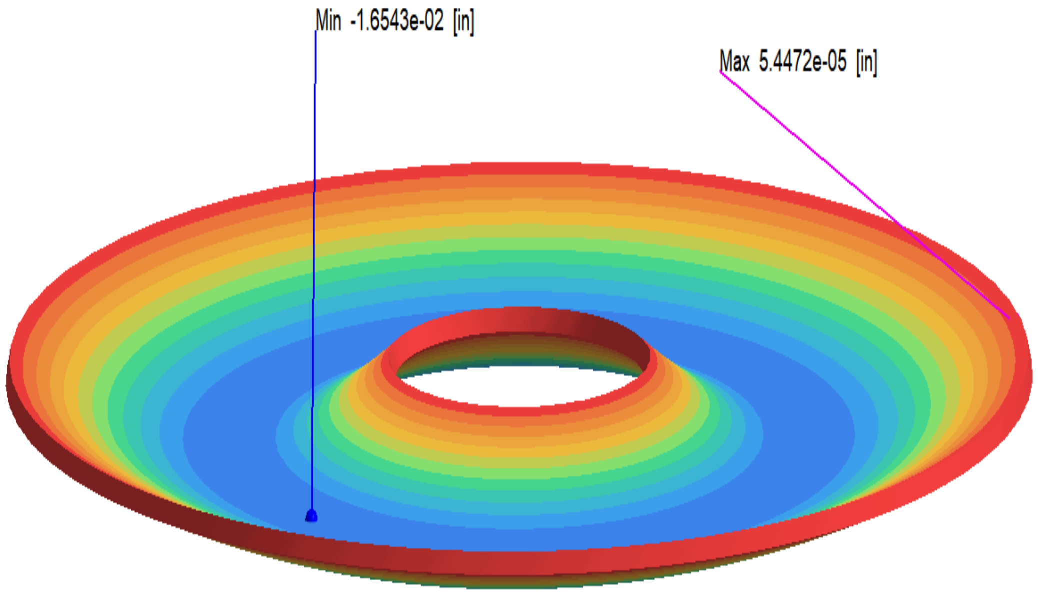
$$M_r = 115.45\ lb - in/in$$

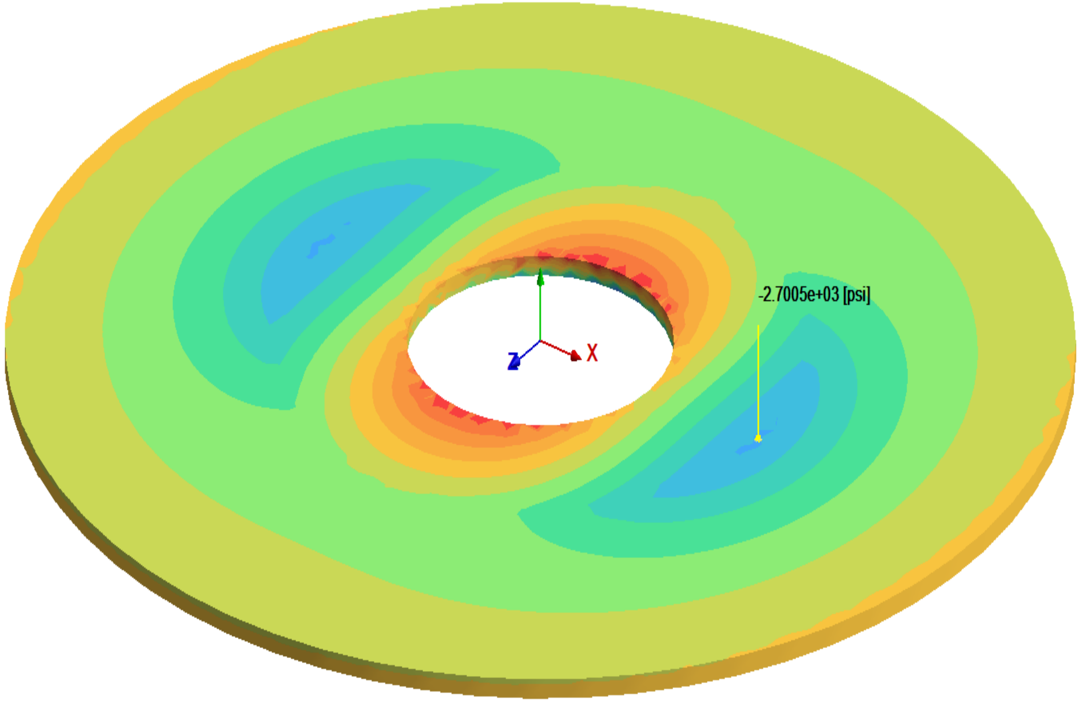
Therefore,

$$Maximum\ bending\ stress = 2772\ psi$$

Analysis Results

Deflection	Bending stress





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Analysis Results

- Maximum Deflection
  - Based on hand calculations – $0.01656\ in$
  - From the simulation – 0.01654 in
- Bending stress
  - Based on hand calculations – $2772\ psi$
  - From the simulation – 2705 psi

Conclusions

- Deformation analysis of annular plate is conducted using Altair SimSolid based on the book listed in slide 2.
- Results of the simulation correlate well to the expected hand calculation value.